EFFECT OF ROUGHNESS ON FRICTIONAL RESISTANCE OF A SURFACE DURING GRADIENT FLOW OF A COMPRESSIBLE GAS WITH HEAT TRANSFER

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In contrast to the large number of methods of calculating the turbulent boundary layer at a smooth surface, the number of known methods of calculating flow over rough surfaces is quite small. At that, they are practically all confined to the case of sand-grain roughness and make it possible to calculate the coefficient of surface friction only for the regime of developed roughness in the absence of a pressure gradient. This situation is connected with the great variety of geometrical forms of roughness and of the means of its distribution over a surface, which makes it difficult to model flow in the vicinity of roughness elements.

In such a case the method of relative correspondence [1] has a definite advantage over other approaches to the solution of the problem. The use of such an approach to the solution problem under consideration makes it possible to extend to the case of surface roughness the possibilities of calculating the turbulent boundary layer using asymptotic relative laws [2].

§1. We introduce the quantity $\Psi_r = (c_{fr}/c_{fs})_{Re_\theta}$, which represents the relative change in the coefficient of friction of a surface owing to the roughness. Here the comparison of the coefficients of friction c_{fr} and c_{fs} at rough and smooth surfaces is made for identical external conditions with the same Reynolds number $Re_\theta = \rho_e \theta u_e/\mu_w$, where θ is the thickness of momentum loss; ρ, u, and μ are the density, velocity, and dynamic viscosity of the gas stream, respectively. Such a choice of the comparison parameter follows from the algorithm, which is general for the theory of relative laws of friction [2], for the solution of the problem when the coefficient of friction of the "standard" surface is calculated at the same value of Re_θ which is obtained from the integral momentum equation for the "perturbed" state of the surface. Here and later the index e pertains to parameters at the external limit of the boundary layer, w to parameters at the wall, aw to parameters at a thermally insulated wall, 1 to parameters at the boundary of the viscous sublayer, s to parameters at a smooth surface, r to parameters at a rough wall, and k to parameters at the line of the tops of the roughness elements.

We obtain the expression for Ψ_r by using a two-layer model for the turbulent boundary layer with the following assumptions:

1. The effect of the longitudinal pressure gradient on the friction at both a smooth and a rough surface is taken into account only through the integral momentum equation, while the direct effect of the pressure gradient on the velocity profile is neglected. As is known, this assumption is approximately correct for flows with negative and moderately positive pressure gradients.

2. The velocity profile in the viscous sublayer is determined from the equation $\tau = \mu_w du/dy = \tau_w$, where $\tau_w = \rho_w u_*^2 = c_f \rho_e u_e^2/2$ is the frictional stress at the wall; u_* is the dynamic velocity. From this we have the velocity profile in the viscous sublayer

$$u^+ = y^+ \quad \text{for} \quad y^+ \leqslant y_1^+; \tag{1.1}$$

where $y^+ = y u_*/\nu_w$ is the distance from the wall; $u^+ = u/u_*$ is the velocity in the variables of the wall law; ν is the kinematic viscosity of the gas.

The thickness $y_{1s}^+ = y_{1s} u_*/\nu_w$ of the viscous sublayer at a smooth surface is assumed to be the same as in a nongradient stream of incompressible liquid: $y_{1s}^+ = 11.6$. In the case of a rough surface the dependence of the thickness of the viscous sublayer on the height $k^+ = k u_*/\nu_w$ of the roughness protuberances (k is the geometrical height) and on their shape is obtained below.

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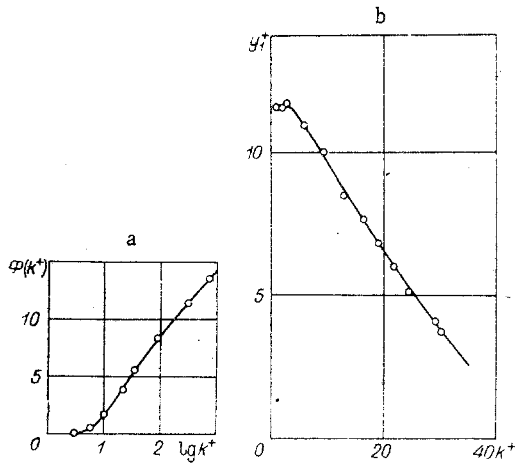

Fig. 1

3. In the turbulent core of the boundary layer (over its entire thickness) we adopt the following distributions of the shear stress τ and the mixing length l, which are valid only in the region near the wall:

$$\tau = \rho l^2 \left(\frac{du}{dy}\right)^2 = \tau_w, \quad l = \varkappa y, \tag{1.2}$$

where $\varkappa = 0.4$ is the turbulence constant.

We integrate (1.2) within the limits of the turbulent core, using a modified Crocco integral for the density distribution

$$\rho_w/\rho = 1 - \alpha\bar{u} - \beta\bar{u}^2. \tag{1.3}$$

Here $\bar{u} = u/u_e$; $\alpha = 1 - T_{aw}/T_w$ is the heat-transfer factor; $T_{aw} = T_e\,[1 + r((\gamma - 1)/2)M_e^2]$ is the temperature of the thermally insulated surface; r is the coefficient of restoration; γ is the ratio of specific heat capacities; M is the Mach number; $\beta = r[(\gamma - 1)/2]M_e^2(T_e/T_w)$.

Using (1.2) and (1.3) we obtain the profile in the form of the velocity defect, independent of the state of the surface:

$$\frac{u_e^+}{\sqrt{\beta}}\left(\arcsin\frac{2\beta\bar{u} + \alpha}{\sqrt{\alpha^2 + 4\beta}} - \arcsin\frac{2\beta + \alpha}{\sqrt{\alpha^2 + 4\beta}}\right) = \frac{1}{\varkappa}\ln\xi \quad \text{for} \quad \xi \geqslant \xi_{\min}, \tag{1.4}$$

where $\xi = y/\delta$; δ is the thickness of the boundary layer; $u_e^+ = u_e/u_* = \sqrt{(2/c_f)(\rho_w/\rho_e)}$ is the friction parameter.

The lower boundary of applicability of the profile (1.4) in the case of a smooth surface is the boundary $y_{\min s} = y_{1s}$ of the viscous sublayer. For a rough surface such a situation ($y_{\min} = y_1$) is retained only so long as the roughness elements are submerged in the viscous sublayer. But if $k \geq y_1$, then the line of the tops of the roughness elements must be taken as the lower boundary, $y_{\min} = k$. Using these considerations and being confined to the first term in the arcsin expansion by powers of the argument, from (1.4) we obtain the expression for Ψ_r:

$$\Psi_r = \left(\frac{\varkappa u_{1s}^+\sqrt{\dfrac{4\beta}{\alpha^2 + 4\beta}} - \ln\xi_{1s}}{\varkappa u_{\min}^+\sqrt{\dfrac{4\beta}{\alpha^2 + 4\beta}} - \ln\xi_{\min}}\right)^2_{\mathrm{Re}_\theta}, \tag{1.5}$$

where, as discussed above,

$$u_{\min}^+ = \frac{u_{\min}}{u_*} = \begin{cases} u_1^+ & \text{for} \quad y_1 \geqslant k \\ u_k^+ & \text{for} \quad k \geqslant y_1, \end{cases} \qquad \xi_{\min} = \begin{cases} \xi_1 & \text{for} \quad y_1 \geqslant k \\ \dfrac{k}{\delta} & \text{for} \quad k \geqslant y_1. \end{cases}$$

Let us convert (1.5) to a form which does not depend explicitly on the relationship between k and y_1. For this purpose we write the velocity profile in the core of the layer in the form of the wall law

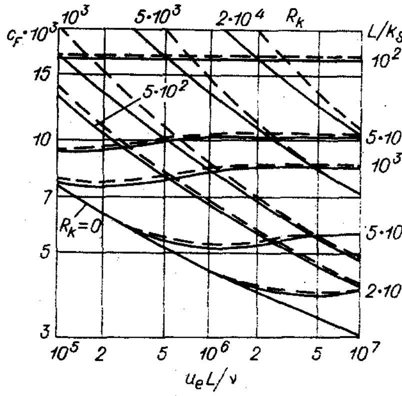

Fig. 2

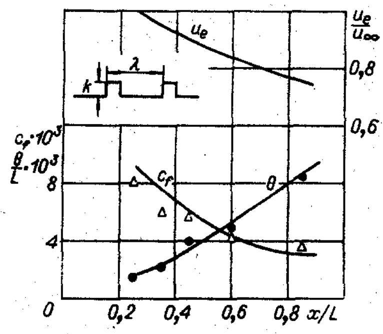

Fig. 3

$$\frac{u_e^+}{\sqrt{\beta}}\left(\arcsin\frac{2\beta\overline{u}+\alpha}{\sqrt{\alpha^2+4\beta}}-\arcsin\frac{\alpha}{\sqrt{\alpha^2+4\beta}}\right)=\frac{1}{\varkappa}\ln y^+ + C-\Phi(k^+,\sigma_1,\sigma_2,\ldots), \tag{1.6}$$

where C = 5.5 is the integration constant; $\Phi(k^+, \sigma_1, \sigma_2, \ldots)$ is the roughness function (Φ = 0 for hydraulically smooth flows); $\sigma_1, \sigma_2, \ldots$ are parameters characterizing the shape and distribution of the roughness elements. The form of the function $\Phi(k^+)$ for the case of sandy roughness is presented in Fig. 1a. The effect of the density of the distribution of other forms of uniform roughness over the surface (two-dimensional elements, elements with a "regular" geometry, etc.) on the behavior of the function $\Phi(k^+, \sigma_1, \sigma_2, \ldots)$ has been studied in a number of experimental reports and has recently been generalized in the form of a unified correlation function [3].

From the experimental results of [4, 5], obtained with a zero pressure gradient, it follows that in both the compressible adiabatic case and for flow with heat transfer the function $\Phi(k^+, \sigma_1, \sigma_2, \ldots)$ retains the same form as in an incompressible stream. The experiments of [6], conducted in an incompressible stream, showed that the roughness function also remains unchanged in the presence of a moderate positive pressure gradient. In such a case it is reasonable to make another assumption.

4. The form of the roughness function $\Phi(k^+, \sigma_1, \sigma_2, \ldots)$ is invariant relative to the total action of the perturbing factors (compressibility, heat exchange, pressure gradient).

Let us join the velocity distribution (1.1) in the viscous sublayer with the distribution (1.6) in which, as before, we are confined to only the first term of the arcsin expansion by powers of the argument. As a result, we obtain the dependence of the thickness y_1^+ of the viscous sublayer on the height k^+ of the roughness

$$y_1^+\sqrt{\frac{4\beta}{\alpha^2+4\beta}}-\frac{1}{\varkappa}\ln y_1^+ = C-\Phi(k^+,\sigma_1,\sigma_2,\ldots). \tag{1.7}$$

In Fig. 1b this dependence is presented in explicit form for the case of an adiabatic surface covered by sandy roughness. Since the possibility of using the two-layer model is limited below by the value $y_1^+ = 1/\varkappa$, here the degeneration of the viscous sublayer is traced only to the value $k^+ \approx 35$. It is well known, however, that the regime of developed roughness corresponding to complete degeneration of the viscous sublayer sets in at $k^+ \approx 70$ for this type of roughness.

It is seen that Eq. (1.7) is connected with the expression for the numerator of (1.5) and that for its denominator in the case when $y_1^+ \geq k_1^+$. But if $k^+ \geq y^+$ then the velocity $u^+_{min} = u^+_k$ can be obtained in a first approximation by extrapolating the velocity distribution (1.6) down to $y^+ = k^+$, which reduces the denominator in (1.5) to the same form:

$$\varkappa u_k^+\sqrt{\frac{4\beta}{\alpha^2+4\beta}}-\ln\left(\frac{k}{\delta}\right)=\ln\delta^+ + \varkappa C-\varkappa\Phi(k^+,\sigma_1,\sigma_2,\ldots).$$

Finally, for Ψ_r we have

$$\Psi_r=\left(\frac{\varkappa C+\ln\delta_s^+}{\varkappa C+\ln\delta_r^+ - \varkappa\Phi(k^+,\sigma_1,\sigma_2,\ldots)}\right)^2_{\mathrm{Re}\theta}. \tag{1.8}$$

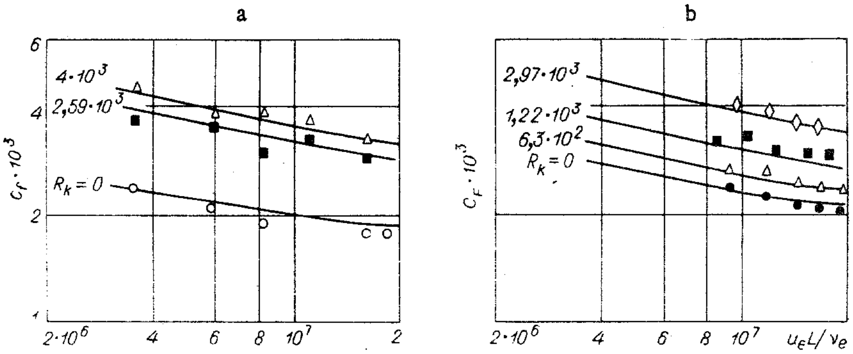

Fig. 4

The roughness of a surface is assigned, as a rule, through the quantity $\mathrm{Re}_k = ku_e/\nu_e$. In such a case the parameters entering into (1.8) are represented in the form

$$k^+ = \mathrm{Re}_k \frac{\mu_e}{\mu_w}\sqrt{\frac{c_{fr}}{2}\frac{\rho_w}{\rho_e}}, \quad \delta^+ = \frac{\mathrm{Re}_\theta}{\theta/\delta}\sqrt{\frac{c_f}{2}\frac{\rho_w}{\rho_e}}. \tag{1.9}$$

An expression can be obtained for $\frac{\theta}{\delta} = \int_0^1 \frac{\rho}{\rho_e}\bar{u}(1-\bar{u})\left(\frac{d\bar{u}}{d\xi}\right)^{-1} d\bar{u}$ if one uses the velocity profile (1.4) over the entire thickness of the boundary layer and not only in its core (the error from such an approximation is slight for large values of the friction parameter u_e^+):

$$\frac{\theta}{\delta} = \varkappa u_e^+ \frac{\rho_w}{\rho_e} \exp\left(-\frac{\varkappa u_e^+}{\sqrt{\beta}} \arcsin\frac{2\beta+\alpha}{\sqrt{\alpha^2+4\beta}}\right) \int_0^1 \frac{\bar{u}(1-\bar{u})}{(1-\alpha\bar{u}-\beta\bar{u}^2)^{3/2}} \exp\left(\frac{\varkappa u_e^+}{\sqrt{\beta}} \arcsin\frac{2\beta\bar{u}+\alpha}{\sqrt{\alpha^2+4\beta}}\right) d\bar{u}. \tag{1.10}$$

To calculate the integral entering into (1.10) we use the fact that $u_e^+ \gg 1$. In this case the integral can be represented in the form of an alternating-sign asymptotic series by powers of $(\varkappa u_e^+)^{-1}$, obtained as a result of integration by parts. Being confined to the first two terms of the series, we have a single expression for θ/δ for smooth and rough surfaces:

$$\frac{\theta}{\delta} = \frac{\sqrt{1-\alpha-\beta}}{\varkappa u_e^+} - \frac{2-\frac{\alpha}{2}+\beta}{(\varkappa u_e^+)^2} = \frac{1}{\varkappa}\sqrt{\frac{c_f}{2}} - \frac{c_f}{2\varkappa^2}\left(2-\frac{\alpha}{2}+\beta\right)\frac{T_w}{T_e}. \tag{1.11}$$

Here and in (1.9) $c_f = c_{fs}$ for a smooth surface and $c_f = c_{fr} = \Psi_r c_{fs}$ for a rough surface, with the values of Ψ_r and c_{fs} being calculated at the same value of Re_θ = idem. It should be noted that at $c_f = c_{f\max} = 2(\varkappa T_e/2T_w(2-\alpha/2+\beta))^2$ the expression (1.11) reaches its maximum and with a further increase in c_f it begins to decline, which does not correspond to reality. Therefore, for the ratio θ/δ in the case when $c_f \ge c_{f\max}$ one must use its maximum value $(\theta/\delta)_{\max} = T_e/4T_w(2-\alpha/2+\beta)$.

§2. The calculation of the frictional resistance in each concrete case is connected, according to the ideas of the method of [2], with the integration of the momentum equation. In the process the functions Ψ_f, Ψ_M, and Ψ_t obtained in [2] are used to allow for the pressure gradient, the compressibility, and the nonadiabaticity of the flow, while the function Ψ_r of the roughness effect is calculated from (1.8) at each step of integration by the method of successive approximations, where $\Psi_r = 1$ is taken as the zeroth approximation. The iteration process converges rapidly; for example, in the determination of the coefficient of friction c_{fr} with 1% accuracy the calculation time for one variant on a BÉSM-6 was ∼1 sec. Using (1.8), one can find that the condition of convergence of the process is a reasonable ratio between the size of the integration step and the height of the roughness protuberances. In the general case this condition has an extremely cumbersome form, but as a provisional value we give the following: $k_s/\Delta x < 5$, where Δx is the integration step and k_s is the height of the equivalent sandy roughness.

The coefficient of local friction was calculated in accordance with the expression $c_{fr} = \Psi_r c_{fs}$, where $c_{fs} = \Psi_M \Psi_t \Psi_f c_{fo}$, where the power-law approximation $c_{fo} = B\mathrm{Re}_\theta^{-m}$ was used for the law of friction under "standard" conditions ($m = 0.25$ and $B = 0.0256$ for $\mathrm{Re}_\theta \le 3\cdot10^3$; $m = 1/6$ and $B = 0.0131$ for $\mathrm{Re}_\theta > 3\cdot10^3$).

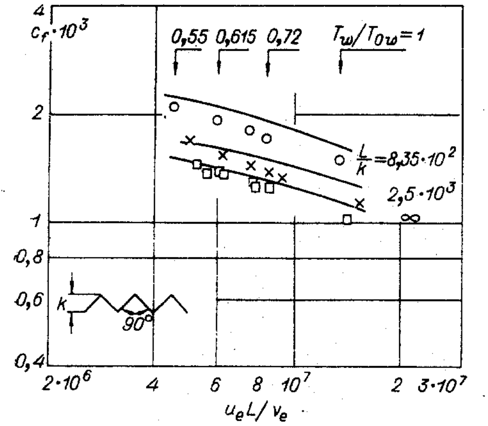

Fig. 5

A comparison between calculations made by the proposed method and the results of measurements of the local and average coefficients of friction c_f and c_F for several forms of uniform roughness is shown in Figs. 2-5 (the form is shown in Figs. 3 and 5). As the roughness function in the calculations we used functions determined in the respective experiments; in all cases they agree well with the correlation equation [3]. In this case, in accordance with assumption 4, to make the calculations it is sufficient to know the behavior of the function $\Phi(k^+, \sigma_1, \sigma_2, \ldots)$ for arbitrary external conditions.

The results of the calculation of the average coefficient of friction of a plate with sandy roughness in an incompressible stream are presented in Fig. 2 (solid lines). The dependences obtained by Prandtl and Schlichting [7] through recalculation to the case of a plate of the results of Nikuradze on the measurement of the resistance of pipes with sandy roughness are plotted with dashed lines. For the family of curves with the parameter L/k the agreement of the results lies within the limits of accuracy of the calculation. Concerning the disagreement in the behavior of the curves with the parameter $Re_k = ku_e/\nu$, it should be noted that the families with the parameters L/k and Re_k presented in [7] do not correlate with each other in the region of difference of the methods being compared. In fact, at the point of intersection of the two curves with the parameters $(L/k)_i$ and $Re_{k,j} = (ku_e/\nu_e)_j$ the equality $(L/k)_i Re_{k,j} = Re_{i,j}$ should be satisfied, where $Re_{i,j} = (u_eL/\nu_e)_{i,j}$ is the Reynolds number at which this intersection occurs. It is seen that this condition is not observed for the dashed lines. As for the behavior of the curves for the local coefficient of friction (graph not presented), here complete agreement with the calculations of Prandtl and Schlichting is observed, with both families in [7] correlating with each other this time.

A comparison with the experimental data of [6], obtained in an incompressible stream with a pressure gradient (the velocity of the outer stream at the initial cross section x/L = 0.25 equals u_∞ = 33.53 m/sec) for a plate (L = 6.096 m) covered by two-dimensional roughness (strips of square cross section with a side k = L/1920 arranged across the stream with a spacing λ = 4k) is presented in Fig. 3.

A comparison with the results of measurements of the local and average coefficients of friction c_f and c_F [4] in a compressible gas stream at a thermally insulated plate with sandy roughness is presented in Fig. 4a, b (M_e = 2 and 2.23, respectively).

The results of a calculation of the friction in a compressible stream (M_e = 4.93) at a plate (L = 0.317 m) with heat exchange are compared in Fig. 5 with the experimental data of [5]. The roughness consists of V-shaped grooves with an angle of 90° at the top and oriented perpendicular to the stream. The heat transfer was provided through variation of the stream temperature (the plate temperature was kept equal to T_w = 305°K), so that the heat-transfer factor was uniquely connected with the Reynolds number (T_w/T_{aw} = 0.55-1).

In all the cases presented above the departure of the calculated values of the coefficients of friction from the experimental values lies within the limits of the experimental accuracy (does not exceed 10%). It should be noted that the accuracy of the method cannot be estimated in advance for flow conditions not tested experimentally.

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ANALYSIS OF THE SOUND FIELD OF A LARGE SPAN OSCILLATING BODY OF REVOLUTION

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The theoretical investigation of the spatial sound field produced by an oscillating body of nonzero thickness is a complex problem which has been solved in practice only for a sphere [1].

In this paper an approximate method is proposed for the analysis of the spatial sound field produced by a slender body of revolution with an arbitrary law of its surface oscillation. The solution obtained can be applied to the analysis of the near sound field and the apparent masses of bodies of revolution oscillating in a compressible fluid.

Let us consider the problem of oscillations of a body of revolution in an ideal compressible fluid, which is at rest infinitely far from the body. Let us introduce the Oxyz Cartesian coordinate system in which the Ox axis is directed along the axis of body symmetry and the origin is at its midsection (see Fig. 1).

Let S be the surface of the undeformed body, $r = \sqrt{y^2 + z^2}$, $r = R(x)$ is the equation of the generator of the body of revolution, $R_0 = R(0)$, l is half the length of the body, $\lambda = l/R_0$ is the span of the body, ω is the angular frequency of body oscillation, t is the time, $\theta = \arctan(z/y)$, $w(x, \theta, t)$ is the displacement of the body surface along the normal to S, a is the speed of sound in the fluid at rest, and $\varphi(x, y, z, t)$ is the velocity potential.

Let us also assume that

$$\lambda \gg 1,\ dR/dx \sim R_0/l; \tag{1}$$

$$|w| \ll R_0,\ \partial w/\partial x \sim A/l\ (A = \max|w|). \tag{2}$$

The assumptions (1) and (2) permit the introduction of two small parameters into the considerations:

$$\varepsilon_1 = R_0/l,\ \varepsilon_2 = A/R_0.$$

Let us go over to dimensionless coordinates x, y, z and functions r, R referred to R_0 by retaining their previous notation. Assuming that the body oscillates according to a given harmonic law for an infinitely long time, we represent the function w and the velocity potential φ in the form

$$\begin{aligned} w(x, \theta, t) &= A\,\mathrm{Re}\{W(x, \theta)e^{i\omega t}\}; \\ \varphi(x, y, z, t) &= aR_0\,\mathrm{Re}\{\Phi(x, y, z)e^{i\omega t}\}. \end{aligned} \tag{3}$$

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